AB Calculus Practice Midterm, 30 MCQ Name_____

No calculator Date_____ Period____

Differentiate each function with respect to *x*.

1) $f(x) = \frac{1}{4}x^{-1}$

A) $f'(x) = \frac{1}{4x^2}$

B) $f'(x) = -\frac{1}{2x^2}$

C) $f'(x) = -\frac{1}{4x}$

D) $f'(x) = -\frac{1}{4x^2}$

2) $f(x) = -\frac{5}{4}x$

A) $f'(x) = \frac{5x}{4}$

B) $f'(x) = -\frac{5x}{4}$

C) $f'(x) = 0$

D) $f'(x) = -\frac{5}{4}$

3) $f(x) = \sqrt[5]{4x - 1}$

A) $f'(x) = \frac{1}{5(4x-1)^{\frac{4}{5}}}$

B) $f'(x) = 4$

C) $f'(x) = \frac{4}{5(4x-1)^{\frac{4}{5}}}$

D) $f'(x) = \frac{4}{(4x-1)^{\frac{4}{5}}}$

4) $f(x) = (-3x + 1)^{-4}$

A) $f'(x) = -\frac{3}{(-3x+1)^5}$

B) $f'(x) = -\frac{4}{(-3x+1)^5}$

C) $f'(x) = \frac{12}{(-3x+1)^5}$

D) $f'(x) = -3$

5) $f(x) = \cos 5x^4$

A) $f'(x) = 20x^3 \sin 5x^4$
B) $f'(x) = -20x^3 \csc 5x^4$
C) $f'(x) = -20x^3 \cot 5x^4$
D) $f'(x) = -20x^3 \sin 5x^4$

6) $f(x) = \sin(-5x^2 + 4)^2$

A) $f'(x) = -20x \tan(-5x^2 + 4)^2 \cdot (-5x^2 + 4)$
B) $f'(x) = 20x \cos(-5x^2 + 4)^2 \cdot (-5x^2 + 4)$
C) $f'(x) = -20x \sec(-5x^2 + 4)^2 \cdot (-5x^2 + 4)$
D) $f'(x) = -20x \cos(-5x^2 + 4)^2 \cdot (-5x^2 + 4)$

7) $f(x) = e^{5x^3}$

A) $f'(x) = 15x^2$
B) $f'(x) = e^{5x^3}$
C) $f'(x) = 15x^2 e^{5x^3}$
D) $f'(x) = 15x^2 e^{5x^3 - 1}$

8) $f(x) = \ln 2x^2$

A) $f'(x) = \dfrac{1}{2x^2}$
B) $f'(x) = 2x^2$
C) $f'(x) = \dfrac{2}{x}$
D) $f'(x) = \dfrac{1}{4x}$

9) $f(x) = (3x^4 - 2) \ln 5x^5$

A) $f'(x) = \dfrac{12x^4 + 5}{x}$
B) $f'(x) = \dfrac{12x^4 \ln 5x^5 + 15x^4 - 10}{x}$
C) $f'(x) = \dfrac{5(3x^4 - 2)}{x}$
D) $f'(x) = 120x^2$

10) $f(x) = (2x^2 + 3) \cdot e^{x^3}$

A) $f'(x) = 3x^2 e^{x^3}(2x^2 + 3)$
B) $f'(x) = x(4 + 3xe^{x^3})$
C) $f'(x) = xe^{x^3}(6x^3 + 9x + 4)$
D) $f'(x) = 24x^3 e^{x^3}$

11) $f(x) = \tan e^{x^3}$

A) $f'(x) = 3x^2 \cdot \cos^2 e^{x^3} \cdot e^{x^3}$
B) $f'(x) = 3x^2 \cdot \sin^2 e^{x^3} \cdot e^{x^3}$
C) $f'(x) = 3x^2 \cdot \sec^2 e^{x^3} \cdot e^{x^3}$
D) $f'(x) = -3x^2 \cdot \sec^2 e^{x^3} \cdot e^{x^3}$

12) $f(x) = e^{\tan 5x^4}$

A) $f'(x) = 20x^3 e^{\tan 5x^4} \cdot \sec^2 5x^4$
B) $f'(x) = 20x^3 \cdot \sec^2 5x^4$
C) $f'(x) = 20x^3 e^{\tan 5x^4 - 1} \cdot \sec^2 5x^4$
D) $f'(x) = e^{\tan 5x^4}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*.

13) $x^3 + 4x^2y^2 + 4xy = 5$

A) $\frac{dy}{dx} = \frac{x^3 + 4x^2y^2 + 4xy}{5}$

B) $\frac{dy}{dx} = \frac{-3x^2 - 8xy^2 - 4y}{5}$

C) $\frac{dy}{dx} = \frac{-3x^2 - 8xy^2 - 4y}{8x^2y + 4x}$

D) $\frac{dy}{dx} = \frac{-8x^2y - 4x}{3x^2 + 8xy^2 + 4y}$

14) $-2y^3 + 2 = 5x^3 + 3y^2$

A) $\frac{dy}{dx} = \frac{-2y^3 + 2}{5x^3 + 3y^2}$

B) $\frac{dy}{dx} = \frac{5x^2}{-2y^2 - 2y}$

C) $\frac{dy}{dx} = \frac{15x^2}{5x^3 + 3y^2}$

D) $\frac{dy}{dx} = \frac{-2y^2 - 2y}{5x^2}$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ at the given point.

15) $1 = 5x - 2xy$ at $(-1, 3)$

A) 1 / 2 B) 2

C) -1 D) 1

16) $-x^2 + 5 = x + 3x^2y^3$ at $(1, 1)$

A) -1 / 10 B) -1

C) 1 D) 9 / 4

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

17)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	1	2	2
2	2	1	4	0
3	3	1	2	$-\frac{3}{2}$
4	4	1	1	-1

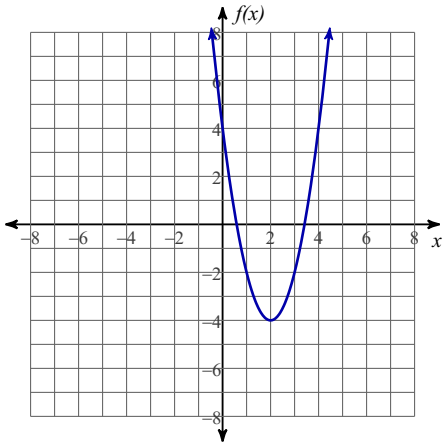
Given $h(x) = f(g(x))$, find $h'(3)$

A) $h'(3) = -\frac{5}{2}$ B) $h'(3) = -\frac{9}{2}$

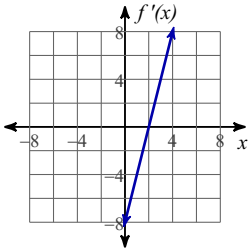
C) $h'(3) = -\frac{3}{2}$ D) $h'(3) = \frac{1}{2}$

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

18)



A)



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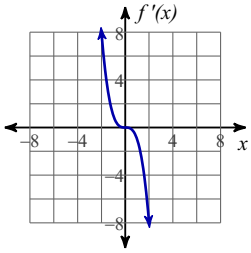

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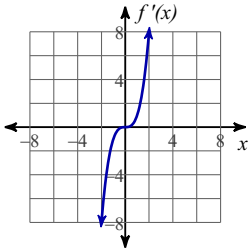
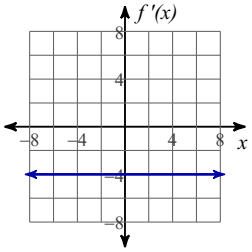


D)



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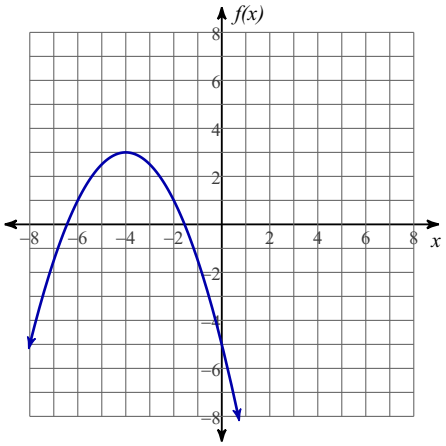
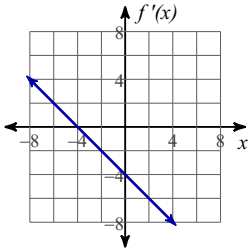
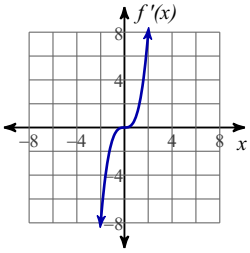


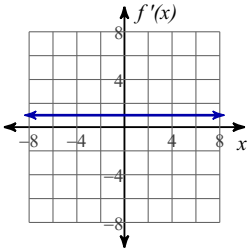
A)



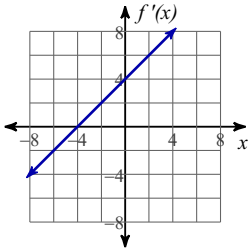
B)



C)



D)



For each problem, find all points of relative minima and maxima.

20) $f(x) = -2\cos(x);\ [-\pi, \pi]$

A) Relative minimum: $(0, -2)$
Relative maxima: $(-\pi, 2), (\pi, 2)$

B) No relative minima.
No relative maxima.

C) Relative minimum: $(0, 1)$
Relative maxima: $(-\pi, -1), (\pi, -1)$

D) Relative minimum: $(0, -1)$
Relative maxima: $(-\pi, 1), (\pi, 1)$

21) $f(x) = x^3 - 3x^2 + 5$

A) Relative minima: $\left(\frac{1}{3}, \frac{127}{27}\right), \left(\frac{2}{3}, \frac{107}{27}\right)$
No relative maxima.

B) No relative minima.
No relative maxima.

C) No relative minima.
Relative maxima: $(4, 21), (8, 325)$

D) Relative minimum: $(2, 1)$
Relative maximum: $(0, 5)$

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

22) $f(x) = -x^3 - 8x^2 - 21x - 16$

A) Inflection point at: $x = -\frac{8}{3}$
Concave up: $\left(-\infty, -\frac{8}{3}\right)$ Concave down: $\left(-\frac{8}{3}, \infty\right)$

B) Inflection point at: $x = -\frac{8}{9}$
Concave up: $\left(-\frac{8}{9}, \infty\right)$ Concave down: $\left(-\infty, -\frac{8}{9}\right)$

C) No inflection points exist.
Concave up: $\left(-\frac{8}{3}, \infty\right)$ Concave down: $\left(-\infty, -\frac{8}{3}\right)$

D) Inflection point at: $x = -\frac{32}{3}$
Concave up: $\left(-\infty, -\frac{32}{3}\right)$ Concave down: $\left(-\frac{32}{3}, \infty\right)$

23) $f(x) = \frac{x^2}{4x - 4}$

A) No inflection points exist.
Concave up: $(1, \infty)$ Concave down: $(-\infty, 1)$

B) Inflection point at: $x = 3$
Concave up: $(4, \infty)$ Concave down: $(-\infty, 4)$

C) Inflection point at: $x = 2$
Concave up: $\left(-\infty, \frac{1}{3}\right)$ Concave down: $\left(\frac{1}{3}, \infty\right)$

D) No inflection points exist.
Concave up: $(-\infty, 1)$ Concave down: $(1, \infty)$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the times t when the particle changes directions.

24) $s(t) = t^3 - 11t^2 + 24t$

A) Changes direction at: $t = \left\{\frac{10}{3}, 12\right\}$

B) Changes direction at: $t = \left\{\frac{22}{3}\right\}$

C) Changes direction at: $t = \left\{\frac{4}{3}, 6\right\}$

D) Changes direction at: $t = \left\{\frac{14}{3}, 14\right\}$

25) $s(t) = -t^3 + 10t^2$

A) Changes direction at: $t = \left\{\frac{11}{3}, 11\right\}$

B) Changes direction at: $t = \left\{\frac{10}{3}, 10\right\}$

C) Changes direction at: $t = \left\{\frac{20}{3}\right\}$

D) Changes direction at: $t = \left\{\frac{13}{3}, 13\right\}$

A particle moves along a horizontal line. Its velocity function is $v(t)$ for $t \geq 0$. For each problem, find the intervals of time when the particle is moving left and moving right and the intervals of time when the particle is slowing down and speeding up.

26) $v(t) = 3t^2 - 8t - 60$

A) Moving left: $\frac{8}{3} < t < 8$, Moving right: $0 \leq t < \frac{8}{3}$, $t > 8$

Slowing down: $0 \leq t < \frac{8}{3}$, $\frac{16}{3} < t < 8$, Speeding up: $\frac{8}{3} < t < \frac{16}{3}$, $t > 8$

B) Moving left: $4 < t < 12$, Moving right: $0 \leq t < 4$, $t > 12$

Slowing down: $0 \leq t < 4$, $8 < t < 12$, Speeding up: $4 < t < 8$, $t > 12$

C) Moving left: $\frac{10}{3} < t < 10$, Moving right: $0 \leq t < \frac{10}{3}$, $t > 10$

Slowing down: $0 \leq t < \frac{10}{3}$, $\frac{20}{3} < t < 10$, Speeding up: $\frac{10}{3} < t < \frac{20}{3}$, $t > 10$

D) Moving left: $0 \leq t < 6$, Moving right: $t > 6$

Slowing down: $\frac{4}{3} < t < 6$, Speeding up: $0 \leq t < \frac{4}{3}$, $t > 6$

Solve each related rate problem.

27) A spherical balloon is deflated at a rate of 36π cm³/sec. At what rate is the radius of the balloon changing when the radius is 2 cm?

A) $-\frac{11}{4}$ cm/sec B) -9 cm/sec

C) $-\frac{9}{2}$ cm/sec D) $-\frac{9}{4}$ cm/sec

28) A spherical snowball melts so that its radius decreases at a rate of 2 in/sec. At what rate is the volume of the snowball changing when the radius is 2 in?

A) -22π in³/sec B) -29π in³/sec

C) -38π in³/sec D) -32π in³/sec

Evaluate each limit using L'Hôpital's Rule.

29) $\lim\limits_{x\to\infty} \frac{\ln x}{2x}$

A) 2 B) 1

C) 0 D) ∞

30) $\lim\limits_{x\to 0} \frac{3\left(e^{x} - e^{-x}\right)}{x}$

A) 6 B) ∞

C) 3 D) 0

Answers to No calculator

1) D
2) D
3) C
4) C
5) D
6) D
7) C
8) C
9) B
10) C
11) C
12) A
13) C
14) B
15) A
16) B
17) C
18) A
19) A
20) A
21) D
22) A
23) A
24) C
25) C
26) D
27) D
28) D
29) C
30) A