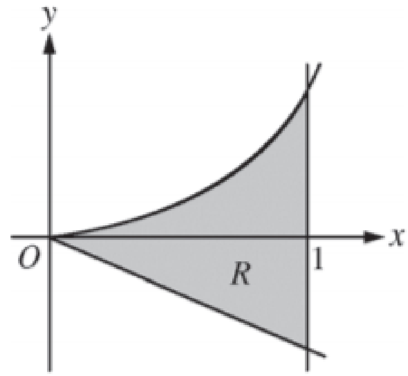
BC Area and Volume FRQs from released AP exams

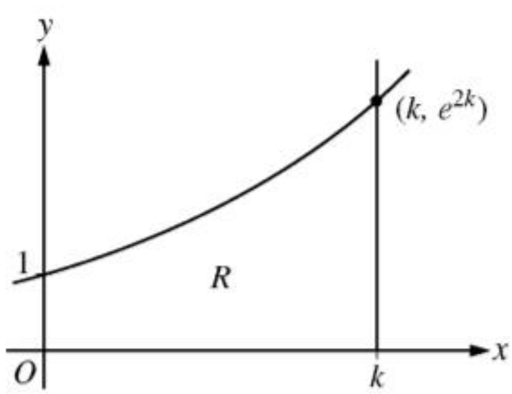


5. Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

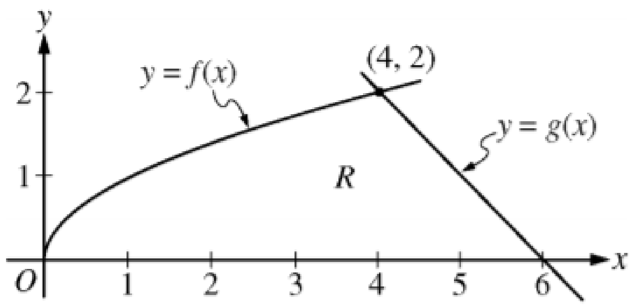


3. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

   (a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

   (b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

   (c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

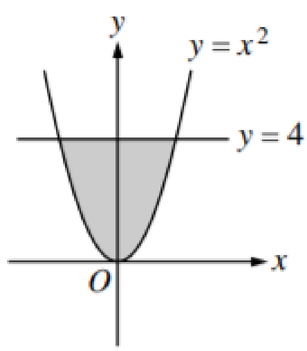


3. The functions f and g are given by $f(x) = \sqrt{x}$ and $g(x) = 6 - x$. Let R be the region bounded by the x-axis and the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) The region R is the base of a solid. For each y, where $0 \leq y \leq 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(c) There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

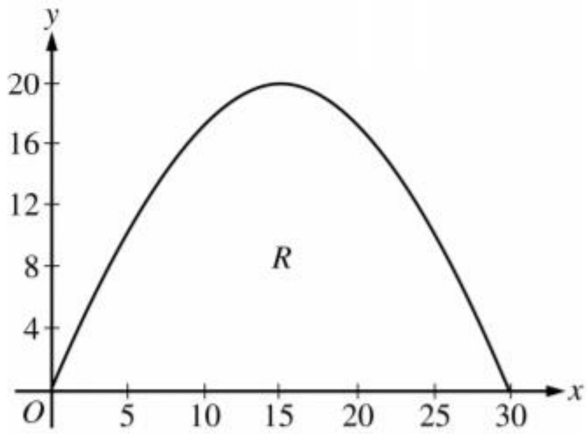


2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated by revolving R about the x-axis.

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

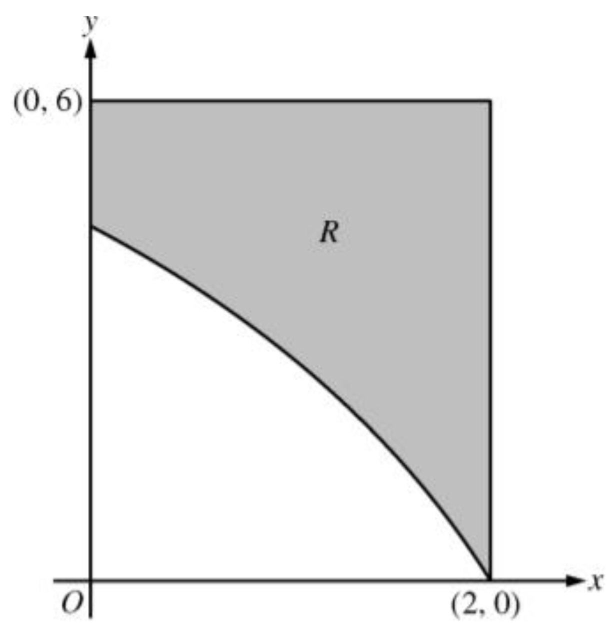


1. A baker is creating a birthday cake. The base of the cake is the region R in the first quadrant under the graph of $y = f(x)$ for $0 \le x \le 30$, where $f(x) = 20\sin\left(\frac{\pi x}{30}\right)$. Both x and y are measured in centimeters. The region R is shown in the figure above. The derivative of f is $f'(x) = \frac{2\pi}{3}\cos\left(\frac{\pi x}{30}\right)$.

   (a) The region R is cut out of a 30-centimeter-by-20-centimeter rectangular sheet of cardboard, and the remaining cardboard is discarded. Find the area of the discarded cardboard.

   (b) The cake is a solid with base R. Cross sections of the cake perpendicular to the x-axis are semicircles. If the baker uses 0.05 gram of unsweetened chocolate for each cubic centimeter of cake, how many grams of unsweetened chocolate will be in the cake?

   (c) Find the perimeter of the base of the cake.



1. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

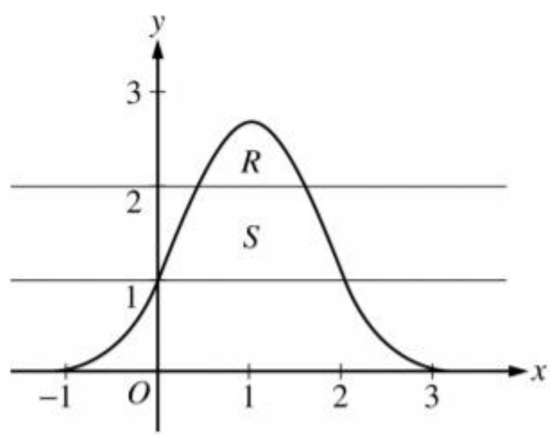
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

4. Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

(a) Find all values of the constant k for which the area of R equals 2.

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

(c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.



1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

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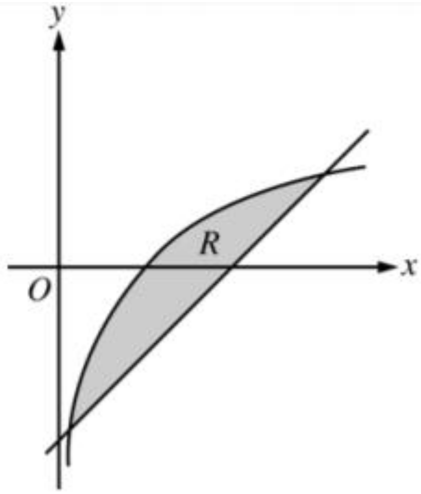
1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

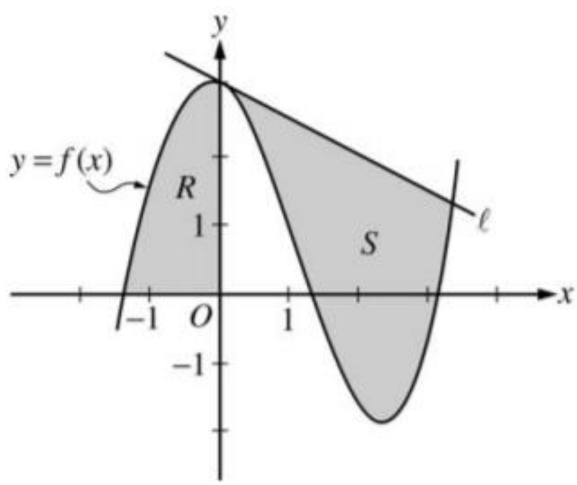


1. Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

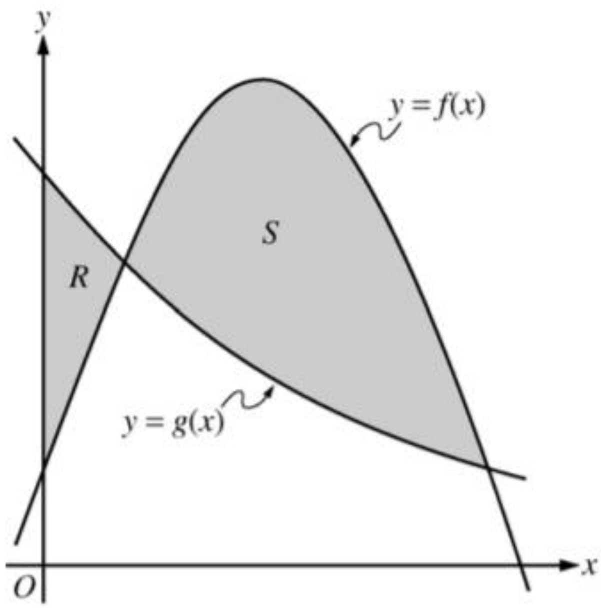


1. Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of S.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

   (a) Find the area of R.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

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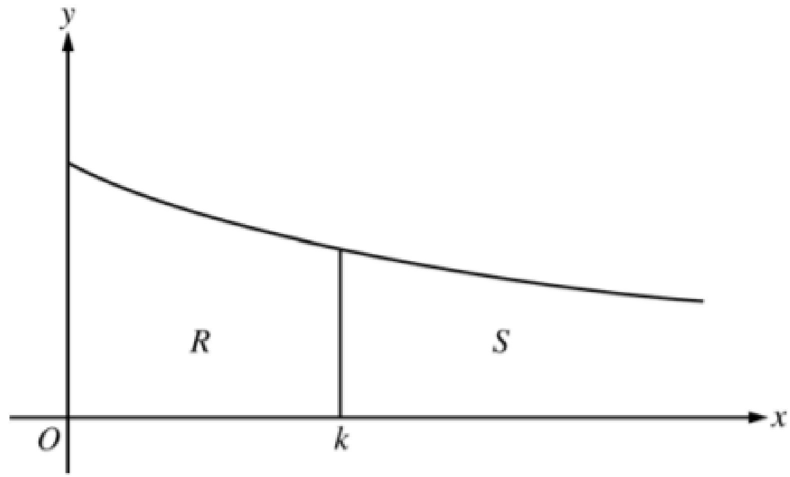
5. Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

   (a) Find the average value of g on the closed interval $[1, 4]$.

   (b) Let S be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the x-axis is revolved about the x-axis. Find the volume of S.

   (c) For the solid S, given in part (b), find the average value of the areas of the cross sections perpendicular to the x-axis.

   (d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b \to \infty} \left[\frac{\int_a^b f(x)\,dx}{b - a} \right]$. Show that the improper integral $\int_4^{\infty} g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

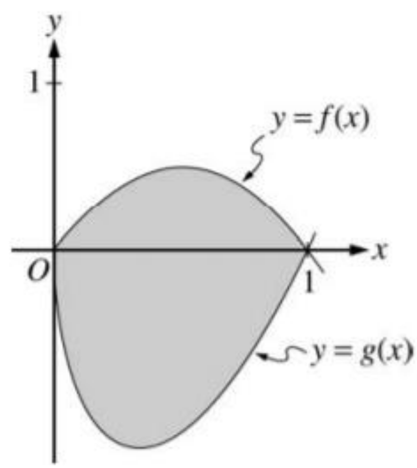


6. Consider the graph of the function f given by $f(x) = \frac{1}{x+2}$ for $x \geq 0$, as shown in the figure above. Let R be the region bounded by the graph of f, the x- and y-axes, and the vertical line $x = k$, where $k \geq 0$.

(a) Find the area of R in terms of k.

(b) Find the volume of the solid generated when R is revolved about the x-axis in terms of k.

(c) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f, as shown in the figure above. Find all values of k such that the volume of the solid generated when S is revolved about the x-axis is equal to the volume of the solid found in part (b).

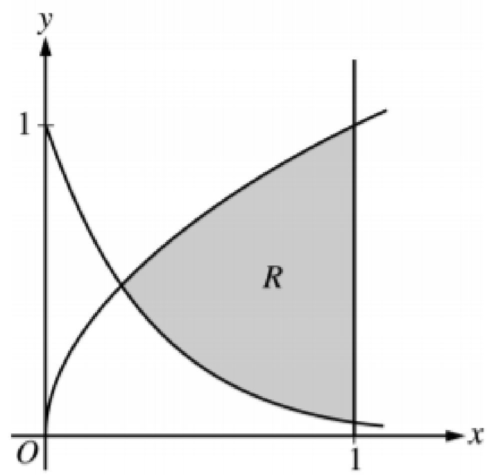


2. Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \leq x \leq 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \leq x \leq 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

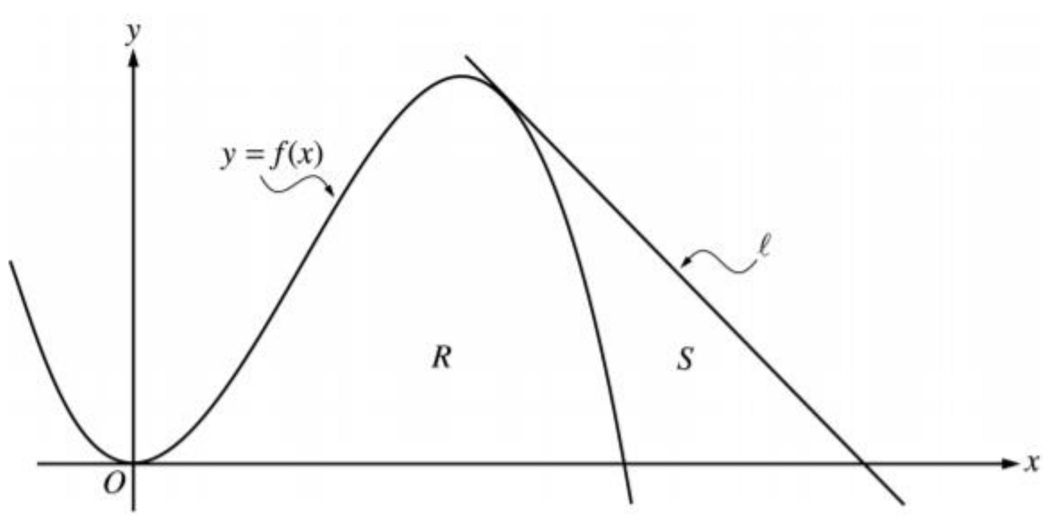


1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

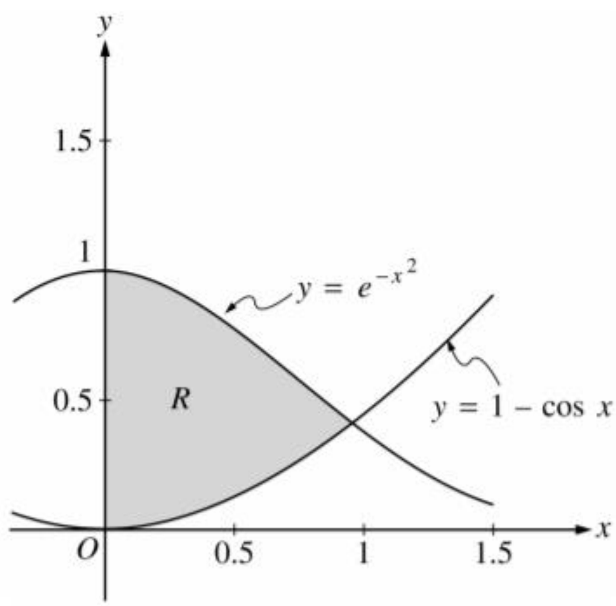
(c) Find the volume of the solid generated when R is revolved about the x-axis.

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

(a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

(b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

(c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.



1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.