

## BC Integration Technique FRQs from released AP exams

5. Let  $f$  be the function defined by  $f(x) = \frac{3}{2x^2 - 7x + 5}$ .

(a) Find the slope of the line tangent to the graph of  $f$  at  $x = 3$ .

(b) Find the  $x$ -coordinate of each critical point of  $f$  in the interval  $1 < x < 2.5$ . Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.

(c) Using the identity that  $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$ , evaluate  $\int_5^{\infty} f(x) dx$  or show that the integral diverges.

(d) Determine whether the series  $\sum_{n=5}^{\infty} \frac{3}{2n^2 - 7n + 5}$  converges or diverges. State the conditions of the test used for determining convergence or divergence.

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4. Let  $f$  be the function defined for  $x > 0$ , with  $f(e) = 2$  and  $f'$ , the first derivative of  $f$ , given by  $f'(x) = x^2 \ln x$ .

(a) Write an equation for the line tangent to the graph of  $f$  at the point  $(e, 2)$ .

(b) Is the graph of  $f$  concave up or concave down on the interval  $1 < x < 3$ ? Give a reason for your answer.

(c) Use antidifferentiation to find  $f(x)$ .

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5. Consider the function  $f(x) = \frac{1}{x^2 - kx}$ , where  $k$  is a nonzero constant. The derivative of  $f$  is given by

$$f'(x) = \frac{k - 2x}{(x^2 - kx)^2}.$$

(a) Let  $k = 3$ , so that  $f(x) = \frac{1}{x^2 - 3x}$ . Write an equation for the line tangent to the graph of  $f$  at the point whose  $x$ -coordinate is 4.

(b) Let  $k = 4$ , so that  $f(x) = \frac{1}{x^2 - 4x}$ . Determine whether  $f$  has a relative minimum, a relative maximum, or neither at  $x = 2$ . Justify your answer.

(c) Find the value of  $k$  for which  $f$  has a critical point at  $x = -5$ .

(d) Let  $k = 6$ , so that  $f(x) = \frac{1}{x^2 - 6x}$ . Find the partial fraction decomposition for the function  $f$ .

Find  $\int f(x) dx$ .

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5. Consider the family of functions  $f(x) = \frac{1}{x^2 - 2x + k}$ , where  $k$  is a constant.

(a) Find the value of  $k$ , for  $k > 0$ , such that the slope of the line tangent to the graph of  $f$  at  $x = 0$  equals 6.

(b) For  $k = -8$ , find the value of  $\int_0^1 f(x) dx$ .

(c) For  $k = 1$ , find the value of  $\int_0^2 f(x) dx$  or show that it diverges.