AP Calculus BC Series Study Guide

Name: ________________________ Date: _______________________

Part I. Geometric Series.

Determine if the following series converge. If they do, find the sum.

$$\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n} \qquad \sum_{n=0}^{\infty} \frac{(-\pi)^{n+1}}{e^{n+2}} \qquad \sum_{n=1}^{\infty} \frac{3^n 10^{-n}}{2^{-n}}$$

Part II. Direct Comparison and Limit Comparison Test.

Match the two series on the left with the two series on the right that should be used by direct comparison to determine convergence or divergence. Use the limit comparison test on the remaining two series on the left (comparing them to the two on the right) to determine convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{n^2}{n^4+n} \qquad \sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{n}{n^2+2} \qquad \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2-2n}$$

$$\sum_{n=1}^{\infty} \frac{3}{n-.5}$$

Part III. Ratio Test.

Use the ratio test to determine convergence or divergence. Typically, the ratio test will come into play when you see a series that has a geometric component, but also has either a factorial or an exponential.

$$\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$$

$$\sum_{n=0}^{\infty} \frac{n!}{5^n}$$

$$\sum_{n=2}^{\infty} \frac{n^2}{(2n-1)!}$$

$$\sum_{n=1}^{\infty} \frac{9^n}{(-2)^{n+1}n}$$

Part IV. Interval of Convergence, Center, and Radius.

When there is a variable embedded in the series, determine the range of values of the variable for which the series would converge. Always check the endpoints of the interval. Sometimes the interval can be determined by looking only at the variable portion of the series; otherwise, the ratio test must be used followed by the limit as n goes to infinity to actually determine which part of the series is the ratio that must be used to determine the interval of convergence (this must ALWAYS be done on a FRQ even if it is apparent at a glance what the ratio portion of the series is).

Find the interval of convergence, center, and radius for each of the following series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$$

$$\sum_{n=1}^{\infty} \frac{n(x-2)^n}{3^{n+1}}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n}(4x-8)^n$$

$$\sum_{n=1}^{\infty} \frac{(x+\frac{1}{3})^n}{2^n n}$$

Part V. The Integral Test.

When the series could be modeled by an integrable function, the integral test works well (and is in fact the easiest way to prove aspects of p-series convergence). Use the integral test to determine convergence or divergence of the following series.

$$\sum_{n=0}^{\infty} ne^{-n^2}$$

$$\sum_{n=1}^{\infty} \frac{1}{n+5}$$