Fundamental Theorem of Calculus Guided Practice

Students need much practice with the Fundamental Theorems of Calculus since a good portion of the AP Exam involves these famous theorems.

Students should be able to:

- Use the fundamental theorem to evaluate definite integrals.
- Use various forms of the fundamental theorem in application situations.
- Calculate the average value of a function over a particular interval.
- Use the other fundamental theorem.

Fundamental Theorem of Calculus

Given $\frac{dy}{dx} = 3x^2 + 2x - 4$ with the initial condition $y(2) = -1$. Find $y(3)$.

Method 1: Integrate $y = \int (3x^2 + 2x - 4)\,dx$, and use the initial condition to find C. Then write the particular solution, and use your particular solution to find $y(3)$.

Optional Method: Use the Fundamental Theorem of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Sometimes there is no antiderivative so we must use the optional method and our graphing calculator.

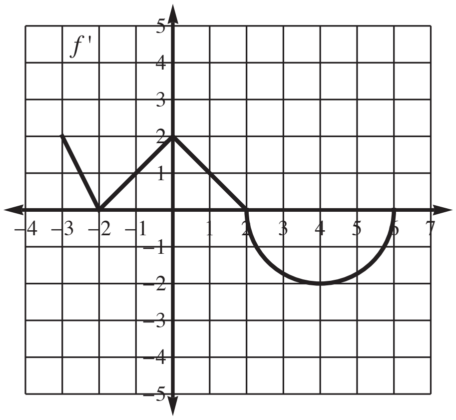
Example 1: $f'(x) = \sin(x^2)$ and $f(3) = -5$. Find $f(2)$.

Example 2 The graph of f' consists of two line segments and a semicircle as shown on the right. Given that $f(-2)=5$, find:

(a) $f(0)$

(b) $f(2)$

(c) $f(6)$



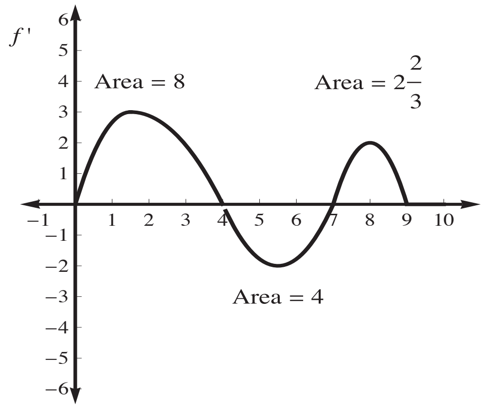
Graph of f'

Example 3 The graph of f' is shown. Use the figure and the fact that $f(4)=5$ to find:

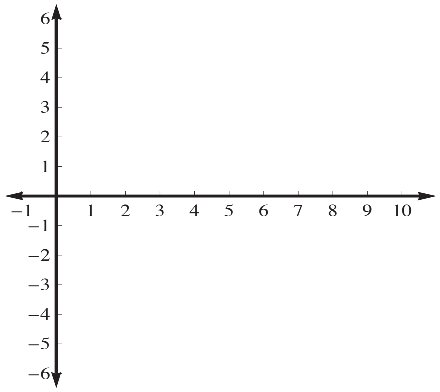
(a) $f(0)$

(b) $f(7)$

(c) $f(9)$



Then sketch the graph of f.



Example 4. A pie with a temperature of 95°C is put into a 25°C room when $t=0$. The pie's temperature is decreasing at a rate of $r(t)=6e^{-0.1t}$ °C per minute. Estimate the pie's temperature when $t=5$ minutes.

Fundamental Theorem of Calculus Practice

Work problems 1 - 2 by both methods. Do not use your calculator.

1. $y' = 3 - \frac{1}{x^2}$ and $y(1) = 6$. Find $y(2)$.

2. $f'(x) = \cos(3x)$ and $f(0) = 3$. Find $f\left(\frac{\pi}{4}\right)$.

Work problems 3 – 6 using the Fundamental Theorem of Calculus and your calculator.

3. $f'(x) = \cos(x^3)$ and $f(0) = 4$. Find $f(1)$.

4. $f'(x) = e^{-x^2}$ and $f(4) = 3$. Find $f(2)$.

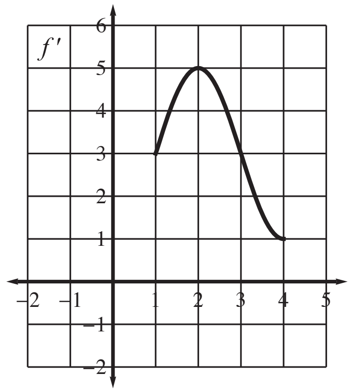
5. A particle moving along the x-axis has position $x(t)$ at time t with the velocity of the particle $v(t) = 5\cos(t^2)$. At time $t = 5$, the particle's position is (3, 0). Find the position of the particle when $t = 7$.

6. Let $F(t)$ represent a bacteria population which is 3 million at time $t = 0$. After t hours, the population is growing at an instantaneous rate of 2^t million bacteria per hour. Find the total increase in the bacteria population during the first four hours, and find the population at $t = 4$ hours.

Use the Fundamental Theorem of Calculus and the given graph.

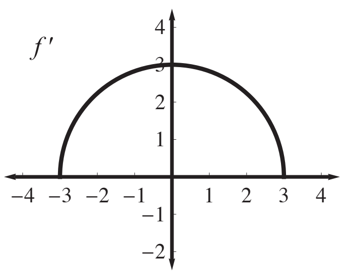
7. The graph of f' is shown on the right.

$\int_1^4 f'(x)\,dx = 6.2$ and $f(1) = 3$. Find $f(4)$.



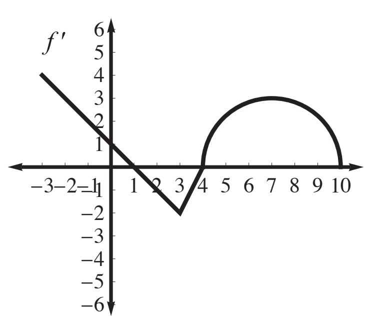
8. The graph of f' is the semicircle shown on the right.

Find $f(-3)$ given that $f(3) = 9$.



9. The graph of f', consisting of two line segments and a semicircle, is shown on the right. Given that $f(-3) = 6$, find:

(a) $f(1)$ (b) $f(4)$ (c) $f(7)$



Fundamental Theorem of Calculus Guided Practice Solutions

Given $\frac{dy}{dx} = 3x^2 + 2x - 4$ with initial condition y (2) = -1. Find y(3).

$$y = \frac{3x^3}{3} + \frac{2x^2}{2} - 4x + C$$

$$-1 = 8 + 4 - 8 + C$$

Method 1: $-5 = C$

$$y = x^3 + x^2 - 4x - 5^3$$

$$y(3) = 3^3 + 3^2 - 12 - 5 = 19$$

Optional Method: $\int_2^3 (3x^2 + 2x - 4)\,dx = y(3) - y(2)$

$$y(2) + \int_2^3 (3x^2 + 2x - 4)\,dx = -1 + (x^3 + x^2 - 4x)\Big|_2^3$$

$$= -1 + (27 + 9 - 12 - 8 - 4 + 8) = 19$$

Example 1: $\int_2^3 (\sin(x^2)\,dx = f(3) - f(2)$

$$f(2) = f(3) - \int_2^3 (\sin(x^2)\,dx = -5 + .031 = -4.96878$$

Example 2:

a) $\int_{-2}^0 f'(x)\,dx = f(0) - f(-2)$

$\int_{-2}^0 f'(x)\,dx + f(-2) = f(0)$

$\frac{1}{2}(2)(2) + 5 = 7$

b) $\int_{-2}^2 f'(x)\,dx = f(2) - f(-2)$

$\int_{-2}^2 f'(x)\,dx + f(-2) = f(2)$

$5 + 4 = 9$

c) $\int_{-2}^6 f'(x)\,dx = f(6) - f(-2)$

$\int_{-2}^6 f'(x)\,dx + f(-2) = f(6)$; $9 - \frac{1}{2}(\pi)(2^2) = 9 - 2\pi$

Example 3:

a) $\int_0^4 f'(x)\,dx = f(4) - f(0)$

$f(0) = f(4) - \int_0^4 f'(x)\,dx = 5 - 8 = -3$

b) $\int_4^7 f'(x)\,dx = f(7) - f(4)$

$f(7) = f(4) + \int_4^7 f'(x)\,dx = 5 - 4 = 1$

c) $\int_4^9 f'(x)\,dx = f(9) - f(4)$

$f(4) + \int_4^9 f'(x)\,dx = f(9)$

$5 - 4 + \frac{8}{3} = \frac{11}{3}$

Example 4: $95 - \int_0^5 6e^{-.1t}dt = 71.392^0 C$

Fundamental Theorem of Calculus Practice

1. $\int_1^2 \left(3 - \frac{1}{x^2}\right)dx = y(2) - y(1) = \frac{17}{2}$

2. $\int_0^{\frac{\pi}{4}} \left(\cos(3x)\right)dx = f(\frac{\pi}{4}) - f(0) = 3 + \frac{\sqrt{2}}{6}$

3. $\int_0^1 \left(\cos(x^3)\right)dx = f(1) - f(0)$

$4 + \int_0^1 \left(\cos(x^3)\right)dx = 4.932$

4. $\int_2^4 e^{-x^2}dx = f(4) - f(2) = 3 - \int_2^4 e^{-x^2}dx = 2.996$

5. $\int_5^7 v(t)dt \ = s(7) - s(5)$

$\int_5^7 v(t)dt \ + s(5) = s(7) = \int_5^7 5\cos(t^2)\,dt + 3 = 2.734$

6. $\int_0^4 2^t\,dt \ = F(4) - F(0)$

$3 + \int_0^4 2^t\,dt$ =24.640 million bacteria

Fundamental Theorem of Calculus Guided Practice Solutions

7.
$$\int_1^4 f'(x)dx = f(4) - f(1)$$
$$6.2 = f(4) - 3$$
$$f(4) = 6.2 + 3 = 9.2$$

8.
$$\int_{-3}^3 f'(x)\,dx = f(3) - f(-3)$$
$$\frac{1}{2}\pi(3^2) = 9 - f(-3)$$
$$f(-3) = 9 - \frac{9\pi}{2}$$

9. a.
$$\int_{-3}^1 f'(x)dx = f(1) - f(-3)$$
$$\frac{1}{2}(4)(4) = f(1) - 6$$
$$f(1) = 6 + 8 = 14$$

b.
$$\int_{-3}^4 f'(x)dx = f(4) - f(-3)$$
$$f(4) = 6 + 8 - \frac{1}{2}(3)(2) = 11$$

c.
$$\int_{-3}^7 f'(x)dx = f(7) - f(-3)$$
$$f(7) = 6 + 5 + \frac{1}{4}(\pi)(9) = 11 + \frac{9\pi}{4}$$