Probability Packet **Name:** ______________________ **Period:** __________

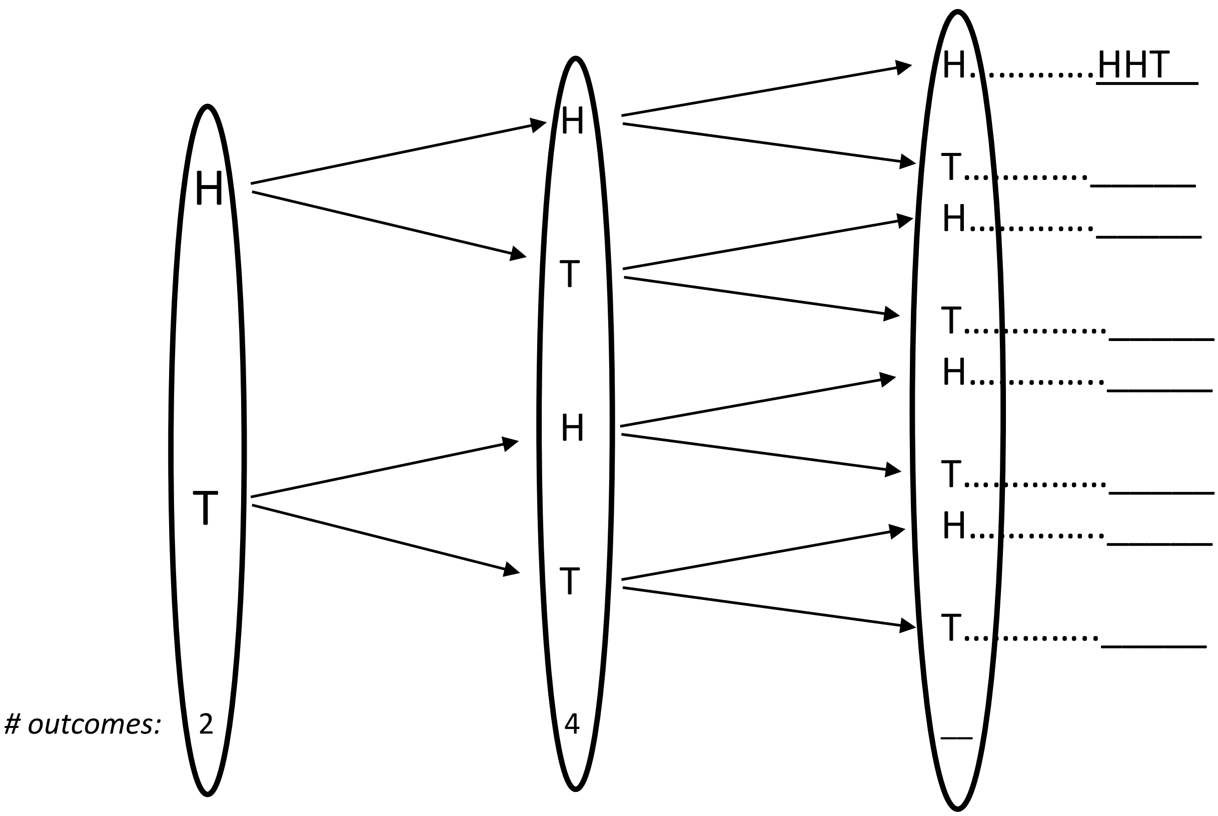
Part 1 – The Fundamental Counting Principle

The **Fundamental Counting Principle** states that the number of possible outcomes of multiple independent events is the equivalent to the number of possible outcomes of one event multiplied by the number of possible outcomes of the other event. Consider the following:

The number of possible outcomes of a single coin flip is two – heads or tails.

What about the number of possible outcomes of two coin flips? The fundamental counting principle states that it will be the number of outcomes of the first even (two) multiplied by the number of outcomes of the second (two), therefore the number of possible outcomes of two coin flips would be four. This can best be understood by thinking of the following probability tree – try filling in the blanks for the 3rd flip:

Flip 1 possible outcomes Flip 2 possible outcomes Flip 3 possible outcomes



What are the total number of possible outcomes for the following:

1 standard die roll? ______________ 2 standard die rolls? _________________

1 standard die roll and 1 coin flip? _______________ 2 die rolls and 2 coin flips? ________________

Picking 1 card from a 52 card deck, 1 die roll, and 1 coin flip? ________________

Making a sandwich from the following options: 3 kinds of bread, 4 kinds of meat, 2 kinds of cheese, and either mustard or mayonnaise? __________________

PERMUTATIONS - Possible arrangements for a certain number of elements/outcomes

Using the **Fundamental Counting Principle** to determine how many different possible combinations there can be from a group of elements or events.

Consider the number of possible ways to arrange 5 people. This is best understood as the number of possible people available to place in the 1st position, then the number of possible people in the 2nd position, etc. Consider drawing a model to help visualize the problem. Fill in the boxes below to represent the number of people available for each of the five positions. Then using the fundamental counting principle to decide the total number of outcomes:

$$\square \times \square \times \square \times \square \times \square = \_\_\_\_\_\_\_\_\_\_\_\_$$

How many different orders (permutations) are there for the following:

4 people sitting around a table: ____________________

3 horses at the end of a horserace: ____________________

FACTORIAL – the mathematical operation used above is called a factorial, and is notated with an exclamation point. Try the following:

1! _______ 2! _________ 6! ___________________ (3!)x(4!) ____________________________

Since the factorial operation is based on the number of ways to order an element or group of elements, even zero can be ordered one way (or can be thought of as being one member of a set containing zero) therefore **0! = 1**

PERMUTATIONS when not all elements in a group are being ordered – "taking 'n' elements 'r' at a time"

Imagine picking the possible combinations of top three finishers in a 100 meter dash out of six runners. Use the same approach as above, but you will be only filling the top three positions:

$$\square \times \square \times \square = \_\_\_\_\_\_\_\_\_\_\_\_$$

Continue this thinking and solve the following possible numbers of combinations:

10 people sitting in 4 chairs: ____________________________

The top two finishers in a horse race out of 8 horses: _______________________________

WHAT IF ORDER DOESN'T MATTER?

Combinations – "taking 'n' elements 'r' at a time, where the order of 'r' doesn't matter"

Consider the problem of the 100 meter dash from the previous page. What if instead of being interested in the possible combinations of 1st, 2nd, and 3rd place finishers, we only wanted to know the possible number of groups of top 3 finishers. If we call the top three finishers runners A, B, C, then any order of these in the top three would be considered one result – ABC represents the same as BCA, etc. So you must think about how many ways the SAME people can end up in the top 3, and divide your answer by that amount do cancel out all of the repeated answers.

Begin with the results from the previous page, then divide by the number of ways of ordering three people:

$$\frac{\boxed{6} \times \boxed{5} \times \boxed{4}}{\boxed{3} \times \boxed{2} \times \boxed{1}} = \frac{120}{3!} = \frac{120}{6} = 20$$

$Note\ -\ another\ way\ of\ thinking\ about\ this\ situation\ is\ \frac{6!}{(6-3)! * 3!}\ which\ gives\ you\ the$ formula $\frac{n!}{(n-r)! * r!}$ where n is the number of elements, and r is the number taken at a time.

Try the following problems:

4 students are being selected from a class of 15 to complete a project. How many combinations of 4 students can be made out of the 15 students, where order doesn't matter?

Selecting a jury of 12 people from a pool of 50, how many possible combinations of 12 can you have where order doesn't matter?

Part 2 – Probability

A **SAMPLE SPACE** is the set of all possible equally likely outcomes from an event. For example, if the event is a single flip of a coin, the sample space would be the set {H,T} for 1 heads and 1 tails. Two flips of a coin would have the following sample space: {HT, TH, HH, TT}.

The **PROBABILITY** of an event is the number of possible outcomes of that even out of the total number of equally likely outcomes. For example, the probability of flipping a heads on one flip of a coin is equal to 1 out of 2 (or ½, or .5, etc.), as there is one item Heads in the sample space for one flip. The probability of flipping two coins and having them both land on heads is equal to 1 out of 4 (or ¼ , .25, etc.) as there are four items in the sample space for two flips, but only one of those outcomes is heads.

Consider the following problem – what is the probability of receiving a head and two tails when flipping a coin three times?

1. Use the fundamental counting principal to decide how many possible outcomes there are:

   2 x 2 x 2 = 8 total outcomes (2 for each flip).

2. Determine the number of possible ways that the desired outcome could occur:

   HTT, THT, TTH or three desirable outcomes

3. Divide the # of desired outcomes by the # of outcomes in the sample space:

$$\frac{3\ desired\ outcomes}{8\ outcomes\ in\ the\ sample\ space} = \frac{3}{8} = .375\ or\ 37.5\%$$

Try the following problems:

1. Determine the probability of rolling:

   a) an 11 on two standard dice

   b) a 5 on two standard dice

2. If a bag of marbles contains 3 black marbles, 2 white marbles, 7 blue marbles, 10 red marbles, and 1 green marble, determine the following probabilities:

   a) pulling a black marble out of the bag on one pull

   b) pulling either a white marble or a blue marble on one pull

   c) not pulling a red marble on one pull

Sometimes it is useful to understand the probability of an outcome by looking at the probability that the outcome does not occur. The probability of a desired outcome NOT happening is called the **COMPLEMENT** of the outcome. For example, if the probability of rolling a 2 on a standard die is 1 in 6, the probability of not rolling a 2 would be the chance of getting any other number, or 5 in 6. Consider what the sum of the probability of a desired outcome and its complement would have to be.

Try the following – determine the probability of the complement of rolling a 7 on the roll of two standard dice:

PROBABILITY OF INDEPENT EVENTS occurring uses the fundamental counting principle. Simply multiply all probabilities together. Consider the probability of each event separately, then multiply each probability. We will return to the marble problem to practice:

If a bag of marbles contains 3 black marbles, 2 white marbles, 7 blue marbles, 10 red marbles, and 1 green marble, determine the following probabilities:

1. Determine the probability of pulling a black marble, then pulling a green marble with replacement (meaning that you return each marble to the bag before pulling again).

2. Determine the probability of pulling two blue marbles in a row with replacement.

3. Determine the probability of NOT pulling a white marble on two pulls, with replacement.

4. Determine the probability of pulling a green marble, then a red marble WITHOUT replacement (meaning that you do NOT return the marble to the bag after each pull).

5. The probability of pulling a black marble, then NOT pulling a green marble without replacement.

PROBABILITY FROM A TABLE. Consider the tables below and calculate each of the probabilities:

The table shows the data for car insurance quotes for 125 drivers made by an insurance company in one week.

	Teen	Adult (20 or over)	Total
0 accidents	15	53	68
1 accident	4	32	36
2+ accidents	9	12	21
Total	28	97	125

You randomly choose one of the drivers. Find the probability of each event.

5. The driver is an adult.

6. The driver is a teen with 0 or 1 accident.

7. The driver is a teen.

8. The driver has 2+ accidents.

9. The driver is a teen and has 2+ accidents.

10. The driver is a teen or a driver with 2+ accidents.

Use the following information for Exercises 11–16. The table shown shows the results of a customer satisfaction survey for a cellular service provider, by location of the customer. In the survey, customers were asked whether they would recommend a plan with the provider to a friend.

	Arlington	Towson	Parkville	Total
Yes	40	35	41	116
No	18	10	6	34
Total	58	45	47	150

11. The customer was from Towson and said No.

12. The customer was from Parkville.

13. The customer said Yes.

14. The customer was from Parkville and said Yes.

15. The customer was from Parkville or said Yes.